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LETTER TO THE EDITOR

Approximations for the fields under steady space-charge perturbed current flow

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Abstract. Working approximations are given for the fields at the electrodes of a film sample in terms of the ratio between the steady excess current and its space charge limit.

One sometimes requires to determine the field distribution in a thin film sample carrying a steady, space-charge perturbed, excess current. For example, one might wish to deduce the field at the injecting electrode when the measured current density is J and the space-charge limit J_{SCL} can be estimated. The field distribution is given by a well known expression (Mott and Gurney 1940, Lampert and Mark 1970) which, for our purpose, is best written as

$$F(\xi) = \tfrac{3}{2}\bar{F}f(\xi + \xi_0)^{1/2}. \tag{1}$$

Here $\xi = x/L$, x being the distance from the injecting electrode and L the film thickness; ξ_0 is a constant of integration; $\bar{F}$ is the average field V/L where V is the applied voltage; and $f = (J/J_{\mathrm{SCL}})^{1/2}$ where $J_{\mathrm{SCL}} = \frac{9}{8}\varepsilon\mu^*\bar{F}^2/\mathrm{L}$ and μ^* represents the (generally) trap-controlled effective carrier mobility, assumed to be independent of field. In particular, the fields at the injecting and counter electrodes are

$$F_0 = \tfrac{3}{2}\bar{F}f\xi_0^{1/2} \qquad F_L = \tfrac{3}{2}\bar{F}f(1 + \xi_0)^{1/2}. \tag{2}$$

The constant ξ_0 must be determined from the condition $\int FL\,\mathrm{d}\xi = V$ which simplifies to

$$(1 + \xi_0)^{3/2} - \xi_0^{3/2} = f^{-1}. \tag{3}$$

For a given value of f, ξ_0 must be found by computation. The author is not aware of any simple approximate solution for ξ_0 in terms of f available in the literature, and the purpose of this letter is to offer such a working approximation.

Let $y = \frac{2}{3}(F_L/\bar{F})f^{-1}$, then (2) and (3) yield

$$y^3 - (y^2 - 1)^{3/2} = f^{-1}. \tag{4}$$

For small enough f, expansion gives

$$y^2 - \tfrac{2}{3}f^{-1}y - \tfrac{1}{4} = 0 \tag{5}$$

which has the approximate solution, linear in $f^2 = J/J_{\mathrm{SCL}}$,

$$F_L/\bar{F} \simeq 1 + \tfrac{9}{16}f^2. \tag{6}$$

For $J/J_{\mathrm{SCL}} \leqslant 0.3$, this approximation differs from the computed values by $<1\%$, but it

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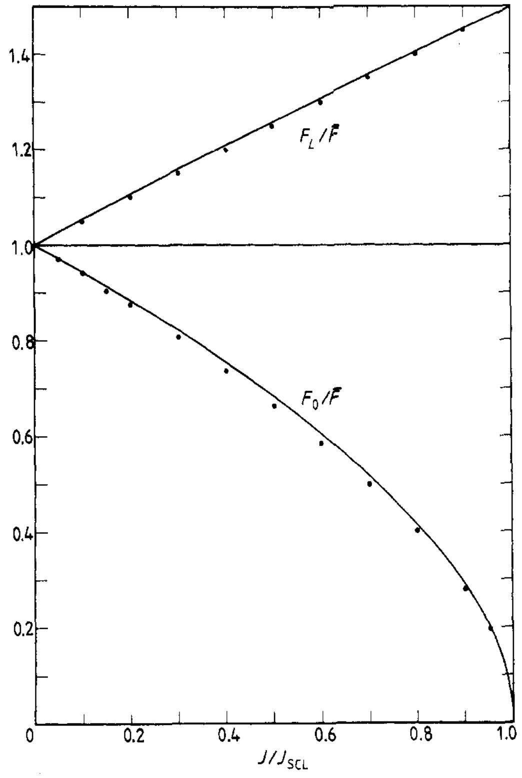

Figure 1. $F_L/\bar{F}$ and $F_0/\bar{F}$ as functions of J/J_{SCL}. Curves: computed; points: calculated from approximations (7) and (9).

clearly fails for larger arguments, since $F_L/\bar{F}$ should tend to 1.5 as J approaches J_{SCL}. However, the *ad hoc* approximation, also linear in J/J_{SCL},

$$F_L/\bar{F} \simeq 1 + 0.5f^2 = 1 + 0.5J/J_{SCL} \tag{7}$$

turns out to be surprisingly good. The error is greatest, but only about −1%, for $J/J_{SCL} = 0.3$, and tends to zero as J/J_{SCL} approaches 0 or 1. From (7) and (2) we then obtain

$$\xi_0 \simeq \tfrac{1}{9}[4(J_{SCL}/J) + (J/J_{SCL}) - 5] \tag{8}$$

and

$$F_0/\bar{F} \simeq [1 - 1.25(J/J_{SCL}) + 0.25(J/J_{SCL})^2]^{1/2}. \tag{9}$$

The errors in ξ_0 and $F/\bar{F}$ amount to about −7.5% and −4% respectively for J/J_{SCL} between 0.7 and 0.9, but decrease towards zero as J/J_{SCL} approaches 0 or 1.

The curves in figure 1 show the computed dependence of $F_L/\bar{F}$ and $F_0/\bar{F}$ on J/J_{SCL}. The points represent values calculated from the approximations (7) and (9), which would seem to afford sufficient accuracy for most practical purposes.

References

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Mott N F and Gurney R W 1940 *Electronic Processes in Ionic Crystals* (Oxford: Clarendon)